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A "littlest Higgs" model with custodial SU(2) symmetry

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Abstract: In this note, a "littlest higgs" model is presented which has an approximate custodial SU(2) symmetry. The model is based on the coset space SO(9)/(SO(5) × SO(4)). The light pseudo-goldstone bosons of the theory include a *single* higgs doublet below a TeV and a set of three $SU(2)_W$ triplets and an electroweak singlet in the TeV range. All of these scalars obtain approximately custodial SU(2) preserving vacuum expectation values. This model addresses a defect in the earlier SO(5) × SU(2) × U(1) moose model, with the only extra complication being an extended top sector. Some of the precision electroweak observables are computed and do not deviate appreciably from Standard Model predictions. In an S-T oblique analysis, the dominant non-Standard Model contributions are the extended top sector and higgs doublet contributions. In conclusion, a wide range of higgs masses is allowed in a large region of parameter space consistent with naturalness, where large higgs masses requires some mild custodial SU(2) violation from the extended top sector.

Keywords: Beyond Standard Model, Higgs Physics.

Contents

1. Introduction

In the near future, experimental tests at the LHC will begin to map out physics at the TeV energy scale. With this data, a determination of the higgs sector, and more importantly, discovering the physics that stabilizes the weak scale from radiative corrections should be achievable goals. However, in the interim, the industry of precision electroweak observables has given us some indirect evidence on what the theory beyond the standard model must look like. And given the unreasonably good fit of the standard model to these observables, these constraints generically suggest a theory with perturbative physics at the TeV scale.

For many years, the only models that could stabilize the weak scale and be weakly perturbative were supersymmetric models, most notably the MSSM. In the past two years, it has been shown that there is a new class of perturbative theories of electroweak symmetry breaking, that of the "little higgs" [1]–[9]. For reviews of the physics, see [10, 11] and for more detailed phenomenology see [12]–[15]. Little Higgs theories protect the higgs boson from one-loop quadratic divergences because each coupling treats the higgs boson as an exact goldstone boson. However, two different couplings together can break the non-linear symmetries protecting the higgs mass, and thus the higgs is a pseudo-goldstone boson with quadratic divergences to its mass pushed to two-loop order. This allows a separation of scales between the cutoff and the electroweak scale, so that physics can be perturbative until the cutoff is reached at $\Lambda \approx 10\,\mathrm{TeV}$.

Having weakly perturbative physics at the TeV scale is probably necessary but definitely not sufficient to guarantee a theory is safe from precision electroweak constraints. Currently precision observables have been measured beyond one-loop order in the standard model, and since little higgs model corrections are parameterically of this order, these observables can put constraints on these theories [16]–[21]. However, these constraints are not unavoidable, and isolating the strongest constraints can point to the necessary features to make little higgs models viable theories of electroweak symmetry breaking. First of all, there are modifications of the original models which address these strongest constraints and greatly ammeliorate the issue [19]. However, just recently, a little higgs model was introduced containing a custodial SU(2) symmetry, the $\mathrm{SO}(5) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ moose model [8]. In the limit of strong coupling for the SO(5) gauge group, the precision electroweak constraints due to the T parameter were softened and in general, there is a large region of parameter space consistent with precision electroweak constraints and naturalness [22].

Let's briefly summarize the physics that gives the custodial SU(2) symmetry. The important point is that in models with a gauged $\mathrm{U}(1) \times \mathrm{U}(1)$ subgroup and standard model fermions gauged under just one of the U(1)'s, the massive B' of these theories provides two constraints. The first constraint is that integrating out the B' generates a custodial SU(2) violating operator that after electroweak symmetry breaking corrects the standard model formula for the mass of the Z gauge boson. This gives corrections to the ρ parameter, and vanishes as the two U(1) gauge couplings become equal. However, the second constraint pulls in the opposite direction in gauge parameter space. This is because the coupling of the B' to standard model fermions generates corrections to low energy four-fermi operators and also to coefficients of the $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ fermion currents. These corrections vanish in the limit in which the U(1) that the standard model fermions is not gauged under becomes strong. Thus, these two constraints prefer different limits in parameter space and can constrain the model.

As pointed out before [8], there are simple modifications that evade these two constraints, such as only gauging $\mathrm{U}(1)_Y$, charging the SM fermions equally under both U(1)'s, or through fermion mixing. Another simple approach that gives custodial SU(2) symmetry is to complete the B' into a custodial SU(2) triplet. If the triplet is exactly degenerate in mass, integrating it out does not contribute to a custodial SU(2) violating operator. To include these new states, instead of gauging two U(1)'s, $\mathrm{SU}(2)_R \times \mathrm{U}(1)$ is gauged. After being broken down to the diagonal $\mathrm{U}(1)_Y$, B' and $W^{r\,\pm}$ are put into a "$\mathrm{SU}(2)_R$" triplet. Integrating out the $W^{r\,\pm}$ generates an operator which only gives mass to the W giving a ρ contribution of the opposite sign of the B' contribution. Numerically, the total ρ contribution from the gauge sector cancels in the strong $\mathrm{SU}(2)_R$ coupling limit (where the triplet becomes degenerate), which is the same limit that reduces corrections to fermion operators.

However, this cancelation is not quite exact for the $SO(5)\times SU(2)\times U(1)$ moose model. The higgs quartic potential of that theory has a flat direction when the two higgs vevs have the same phase, thus viable electroweak symmetry breaking requires the higgs vevs to have different phases. This phase difference changes the ρ contribution due to the $W^{r\,\pm}$ gauge bosons. The higgs currents of the $W^{r\,\pm}$ are not invariant under a vev phase rotation, and thus the cancelation in the strong coupling limit only occurs if the phase is 0 or π. Indeed, this remnant of custodial SU(2) violation puts the strongest constraint on the theory.

The situation can be easily resolved if the little higgs theory contains only a single light higgs doublet. In this case, the $W^{r\,\pm}$ current just transforms by a phase under the vev phase rotation, which cancels out of the contribution. It turns out that the $SO(5)\times SU(2)\times U(1)$ moose's defect can be removed by imposing a $\mathbb{Z}_4$ symmetry inspired by orbifold models [23], which leaves only a single light higgs doublet that still has an order one quartic coupling. In this paper, we will take a different approach and construct a "littlest higgs" model with custodial SU(2) symmetry and just *one* higgs doublet.

This "littlest higgs" model will be based on an $\frac{SO(9)}{SO(5)\times SO(4)}$ coset space, with an $SU(2)_L\times SU(2)_R\times SU(2)\times U(1)$ subgroup of SO(9) gauged. The pseudo-goldstone bosons are a single higgs doublet, an electroweak singlet and a set of three $SU(2)_W$ triplets, precisely the content of one of the original custodial SU(2) preserving composite higgs models [24]. The global symmetries protect the higgs doublet from one-loop quadratic divergent contributions to its mass. However, the singlet and triplets are not protected, and will be pushed to the TeV scale. Integrating out these heavy particles will generate an order one quartic coupling for the higgs. To complete the theory with fermions, the minimal top sector contains two extra colored quark doublets and their charge conjugates.

Since the primary motivation of the model is to improve consistency with precision electroweak observables, the model's corrections to these observables will be calculated. First, we will see that aside from some third generation quark effects, a limit will exist where non-oblique corrections vanish. This limit was recently described as "near-oblique" [21] and we will continue to use this terminology. The existence of this limit allows a meaningful S and T analysis of the oblique corrections, which will be performed in this model to order (v^2/f^2). The dominant contributions come from the extended top sector and the higgs doublet, which are quite mild in most of parameter space. In fact, this analysis will show that there is a wide range of higgs masses allowed in a large region of parameter space consistent with naturalness.

The outline of the rest of the paper is as follows: in section 2 we describe the model's coset space, light scalars and the symmetries that protect the higgs mass. We also analyze the gauge structure and then describe the minimal candidate top sectors. In section 3, we will show how the quartic higgs potential is generated as well as describe the log enhanced contributions to the higgs mass parameter. There will be vacuum stability issues, and we will point out ways which these can be resolved. Also as usual, the top sector contributions will generically drive electroweak symmetry breaking. In section 4, some precision electroweak observables will be calculated and the constraints on the theory will be detailed. In section 5, we conclude and finally in appendix A, we describe our specific generators and representations of SO(4).

2. The model

The first ingredient necessary for custodial SU(2) symmetry is the breakdown of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(2) \times \mathrm{U}(1)$ down to the diagonal $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ subgroup. Therefore the global symmetry group must be at least rank 4. Two rank 4 groups are easy to eliminate — SU(5) does not contain the gauged group and the SO(8) adjoint contains no higgs doublets. This leaves SO(9), Sp(8) and F_4 as the only remaining rank 4 candidates. In this paper, we'll focus on the SO(9) group as it is the easiest to analyze. However, we do mention here that it appears to be difficult to get a single light higgs doublet in the Sp(8), F_4 groups.

Isolating our attention to SO(9), it is straightforward to implement the "little higgs" construction. Using the vector representation, the top four by four block will contain the gauged $\mathrm{SO}(4) \cong \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ and the bottom four by four block will contain the gauged $\mathrm{SU}(2) \times \mathrm{U}(1) \subset \mathrm{SO}(4)$. The coset space should break these two SO(4)'s down to their diagonal subgroup, which can be achieved by an off-diagonal vev for a two-index tensor of SO(9). In order to have the largest unbroken global symmetry (and thus reduce the amount of light scalars), a symmetric two-index tensor should be chosen.

This construction can be described in the following way: take an orthogonal symmetric nine by nine matrix, representing a non-linear sigma model field Σ which transforms under an SO(9) rotation by $\Sigma \to V \Sigma V^T$. To break the SO(4)'s to their diagonal, we take Σ's vev to be

$$\langle \Sigma \rangle = \begin{pmatrix} 0 & 0 & \mathbb{1}_4 \\ 0 & 1 & 0 \\ \mathbb{1}_4 & 0 & 0 \end{pmatrix} \tag{2.1}$$

which breaks the SO(9) global symmetry down to an $\mathrm{SO}(5) \times \mathrm{SO}(4)$ subgroup.[1] This coset space guarantees the existence of $20 = (36 - 10 - 6)$ light scalars. Of these 20 scalars, 6 will be eaten in the higgsing of the gauge groups down to $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$. The remaining 14 scalars consist of a single higgs doublet h, an electroweak singlet ϕ^0, and three triplets ϕ^{ab} which transform under the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ diagonal symmetry as[2]

$$h : (\mathbf{2}_L, \mathbf{2}_R) \qquad \phi^0 : (\mathbf{1}_L, \mathbf{1}_R) \qquad \phi^{ab} : (\mathbf{3}_L, \mathbf{3}_R)\,. \tag{2.2}$$

This spectrum is particularly nice as each set of scalars can have vacuum expectation values that preserve custodial SU(2); we will see later that this is approximately true. These fields parameterize the direction of the Σ field and can be written in the standard way

$$\Sigma = e^{i\Pi/f} \langle \Sigma \rangle e^{i\Pi^T/f} = e^{2i\Pi/f} \langle \Sigma \rangle\,, \tag{2.3}$$

where

$$\Pi = \frac{-i}{4} \begin{pmatrix} 0 & \sqrt{2}\vec{h} & -\Phi \\ -\sqrt{2}\vec{h}^T & 0 & \sqrt{2}\vec{h}^T \\ \Phi & -\sqrt{2}\vec{h} & 0 \end{pmatrix}. \tag{2.4}$$

[1] We could separate the trace from Σ to make it transform as an irreducible representation of SO(9), however this equivalent vev is chosen so that Σ can be orthogonal.

[2] See appendix A for specific representation and generator conventions.

In Π, the would-be goldstone bosons that are eaten in the higgsing down to $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ have been set to zero. The singlet and triplets are contained in the symmetric four by four matrix Φ where

$$\Phi = \phi^0 + 4\phi^{ab}\, T^{l\,a} T^{r\,b}\,. \tag{2.5}$$

It is now simple to determine the global symmetries that protect the higgs mass at one loop. Under the upper five by five $\mathrm{SO}(5)_1$ symmetry, the scalars transform as:

$$\delta\vec{h} = \vec{\alpha} + \cdots \qquad\qquad \delta\Phi = -\frac{1}{2f}\left(\vec{\alpha}\,\vec{h}^{\,T} + \vec{h}\,\vec{\alpha}^{\,T}\right) + \cdots\,. \tag{2.6}$$

Similarly, under the lower five by five $\mathrm{SO}(5)_2$, the scalars transform as:

$$\delta\vec{h} = \vec{\beta} + \cdots \qquad\qquad \delta\Phi = \frac{1}{2f}\left(\vec{\beta}\,\vec{h}^{\,T} + \vec{h}\,\vec{\beta}^{\,T}\right) + \cdots\,. \tag{2.7}$$

Any interaction that preserves at least one of these $\mathrm{SO}(5)$ symmetries treats the higgs as an *exact* goldstone boson. Thus, if all interactions are chosen to preserve one of these symmetries, the higgs mass will be protected from one loop quadratic divergences. In the next two subsections, that motivation is used to determine the requisite interactions of the theory.

2.1 Gauge sector

The gauge group structure obviously follows the preserving $\mathrm{SO}(5)$ symmetry logic. The gauged $\mathrm{SO}(4) \cong \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ is generated by

$$\tau^{l\,a} = \begin{pmatrix} T^{l\,a} & \\ & 0_5 \end{pmatrix} \qquad\qquad \tau^{r\,a} = \begin{pmatrix} T^{r\,a} & \\ & 0_5 \end{pmatrix} \tag{2.8}$$

and preserves $\mathrm{SO}(5)_2$ whereas the gauged $\mathrm{SU}(2) \times \mathrm{U}(1)$ is generated by

$$\eta^{l\,a} = \begin{pmatrix} 0_5 & \\ & T^{l\,a} \end{pmatrix} \qquad\qquad \eta^{r\,3} = \begin{pmatrix} 0_5 & \\ & T^{r\,3} \end{pmatrix} \tag{2.9}$$

and preserves $\mathrm{SO}(5)_1$. The kinetic term for the pseudo-goldstone bosons can now be written as

$$\mathcal{L}_{\mathrm{kin}} = \frac{f^2}{4}\,\mathrm{Tr}\left[D_\mu \Sigma D^\mu \Sigma\right], \tag{2.10}$$

where the covariant derivative is given by

$$D_\mu \Sigma = \partial_\mu \Sigma + i\,[A_\mu, \Sigma] \tag{2.11}$$

with the gauge boson matrix A_μ defined by

$$A \equiv g_L W^{la}_{\mathrm{SO}(4)} \tau^{l\,a} + g_R W^{ra}_{\mathrm{SO}(4)} \tau^{r\,a} + g_2 W^{la} \eta^{l\,a} + g_1 W^{r3} \eta^{r\,3}\,. \tag{2.12}$$

Due to the vev of Σ, the vector bosons mix and can be diagonalized with the following transformations:

$$\begin{aligned} B &= \cos\theta' W^{r3} - \sin\theta' W^{r3}_{\mathrm{SO}(4)} & \qquad B' &= W'^{\,r3} = \sin\theta' W^{r3} + \cos\theta' W^{r3}_{\mathrm{SO}(4)} \\ W^a &= \cos\theta W^{la} - \sin\theta W^{la}_{\mathrm{SO}(4)} & \qquad W'^a &= W'^{\,la} = \sin\theta W^{la} + \cos\theta W^{la}_{\mathrm{SO}(4)}\,, \end{aligned} \tag{2.13}$$

where the mixing angles are related to the couplings by:

$$\begin{aligned}\cos\theta' &= \frac{g'}{g_1} & \sin\theta' &= \frac{g'}{g_R} \\ \cos\theta &= \frac{g}{g_2} & \sin\theta &= \frac{g}{g_L}\,.\end{aligned} \tag{2.14}$$

Notice that there is no relation between θ and θ' since SO(4) has two arbitrary gauge couplings g_L and g_R. They could of course be set equal by imposing a $\mathbb{Z}_2$ symmetry, which we will choose to do when describing the limits on the model. In this L-R symmetric limit, the constraint on the angles in order to get the correct θ_W is $\sin\theta \approx \sqrt{3}\sin\theta'$. The masses for the heavy vectors can now be written in terms of the electroweak gauge couplings and mixing angles:

$$m^2{}_{W'} = \frac{4g^2 f^2}{\sin^2 2\theta} \qquad m^2{}_{B'} = \frac{4g'^2 f^2}{\sin^2 2\theta'} \qquad m^2{}_{W^{r\,\pm}} = \frac{4g'^2 f^2}{\sin^2 2\theta'}\cos^2\theta'\,. \tag{2.15}$$

2.2 Fermion sector

For all fermions besides the top quark, the Yukawa couplings are small, and thus it is not necessary to protect the higgs from their one loop quadratic divergences. However, there is the requirement that low energy observables such as four-fermi operators do not receive large corrections. This can be achieved by gauging the light fermions only under SU(2) $\times$ U(1). In the strong SO(4) coupling limit, these fermions will decouple from the W' and B' and will not give strong precision electroweak corrections.

To implement the Yukawa couplings for the light fermions, we add

$$\mathcal{L}_{LF} = \sqrt{2}f\left[y_u\,(0_4\,u^c\,0_4)\,\Sigma\begin{pmatrix}0_5\\ \vec{q}_u\end{pmatrix} + y_d\,(0_4\,d^c\,0_4)\,\Sigma\begin{pmatrix}0_5\\ \vec{q}_d\end{pmatrix} + y_l\,(0_4\,e^c\,0_4)\,\Sigma\begin{pmatrix}0_5\\ \vec{l}\end{pmatrix}\right] + \text{ h.c.} \tag{2.16}$$

In this expression, we have defined the "SO(4)" representations corresponding to the SU(2) $\times$ U(1) representations by

$$\vec{q}_u \leftrightarrow Q_u = (q\;0_2) \qquad \vec{q}_d \leftrightarrow Q_d = (0_2\;q) \qquad \vec{l} \leftrightarrow L = (0_2\;l)\,, \tag{2.17}$$

where q and l are the standard quark and lepton doublets. The exact correspondence between the two equivalent representations is presented in appendix A. At first order, these interactions reproduce the standard yukawa interactions for the light fermions.

On the other hand, the top yukawa is the strongest one loop quadratic divergence of the standard model and therefore the top sector must be extended in order to stabilize the higgs mass parameter. From the symmetry considerations given earlier, the top sector has to preserve either the $\text{SO}(5)_1$ or $\text{SO}(5)_2$ symmetry. The minimal approach is to preserve the $\text{SO}(5)_1$ symmetry, which can be accomplished by adding t^c to an SO(4) gauge vector $\vec{\mathcal{X}}^c$. In addition to this new vector, we add its charge conjugate $\vec{\mathcal{X}}$ and add a Dirac mass for the two fermions. The interactions are:

$$\mathcal{L}_{\text{top}} = y_1 f\,(\vec{\mathcal{X}}^{c\,T}\,t^c\,0_4)\,\Sigma\begin{pmatrix}0_5\\ \vec{q}_t\end{pmatrix} + y_2 f\vec{\mathcal{X}}^T\vec{\mathcal{X}}^c + \text{ h.c.} \tag{2.18}$$

Now, the choice is whether or not to make $\vec{q}_t$ a "full" SO(4) vector. Since it is only charged under $\mathrm{SU}(2) \times \mathrm{U}(1)$, it does not have to be a full SO(4) vector, but can contain just one doublet like $\vec{q}_u$ above. For the sake of simplicity, we will choose to analyze the most minimal case of one doublet.

In this minimal case, the gauge charges of the fermions are:

$$
\begin{array}{c|ccccc}
 & \mathrm{SU}(3)_c & \mathrm{SU}(2)_L & \mathrm{SU}(2)_R & \mathrm{SU}(2) & \mathrm{U}(1) \\
\hline
q & 3 & 1 & 1 & 2 & 1/6 \\
t^c & \bar{3} & 1 & 1 & 1 & -2/3 \\
\vec{\mathcal{X}} & 3 & 2 & 2 & 1 & 2/3 \\
\vec{\mathcal{X}}^c & \bar{3} & 2 & 2 & 1 & -2/3
\end{array}
\tag{2.19}
$$

Under the diagonal $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$, $\vec{\mathcal{X}}$ contains two doublets $\mathcal{X}_1, \mathcal{X}_2$ with hypercharge 1/6 and 7/6 respectively. Expanding the terms, we find a mass term linking $\mathcal{X}_1^c$ with a linear combination of q and $\mathcal{X}_1$. Integrating out the heavy fermion gives a top yukawa coupling

$$y_t = \frac{y_1 y_2^*}{\sqrt{2(|y_1|^2 + |y_2|^2)}}\,. \tag{2.20}$$

3. Potential and EWSB breaking

By construction, the interactions of the theory do not generate one loop quadratic divergences for the mass parameter of the higgs. To demonstrate this explicitly, the Coleman-Weinberg potential will be computed. The one loop quadratic divergent piece will generate a potential for Φ and h, including a quadratically divergent mass for the Φ. Similar to the $\mathrm{SU}(6)/\mathrm{Sp}(6)$ model [6], the gauge interactions will introduce an instability in the vacuum. The problem is a bit more serious here because the gauge contributions are opposite in sign for the singlet and triplet masses; thus, the origin of the potential is a saddle point. However, as in the $\mathrm{SU}(6)/\mathrm{Sp}(6)$ paper, there are ways to cure this instability issue. Once the instability has been addressed, integrating out the massive Φ will generate an order one quartic coupling for h, but no mass term.

For the log divergent piece of the Coleman-Weinberg potential, we will only analyze the contributions to the higgs mass parameter. As usual, gauge and scalar sectors will give positive contributions whereas the top sector gives a large negative contribution that drives electroweak symmetry breaking.

3.1 One loop quadratic term

The one loop quadratically divergent piece of the Coleman-Weinberg Potential is given by

$$V_{\text{one loop }\Lambda^2} = \frac{\Lambda^2}{32\pi^2}\,\mathrm{Str}(M^\dagger M[\Sigma])\,. \tag{3.1}$$

By the symmetry arguments given earlier, the different $\mathrm{SO}(5)_i$ preserving interactions can generate operators depending on

$$\mathrm{SO}(5)_1: \quad \Phi + \frac{1}{2f}\vec{h}\,\vec{h}^T \qquad \text{or} \qquad \mathrm{SO}(5)_2: \quad \Phi - \frac{1}{2f}\vec{h}\,\vec{h}^T\,. \tag{3.2}$$

It will also be convenient to introduce some notation, where

$$\frac{1}{2f}\vec{h}\,\vec{h}^{\,T} = \mathcal{H}^0 + 4\mathcal{H}^{ab}\,T^{l\,a}T^{r\,b}\,. \tag{3.3}$$

$\mathcal{H}^0$ and $\mathcal{H}^{ab}$ are quadratic in the h fields and their explicit expressions appear in appendix A.

The gauge contribution can be calculated from the kinetic term for Σ, which gives

$$\begin{aligned} V_{\text{gauge}} = & -\frac{9f^2}{8}\left[(g_L^2+g_R^2)(\phi^0-\mathcal{H}^0)^2+\left(g_2^2+\frac{g_1^2}{3}\right)(\phi^0+\mathcal{H}^0)^2\right]+ \\ & +\frac{3f^2}{8}\left[(g_L^2+g_R^2)(\phi^{ab}-\mathcal{H}^{ab})^2+(g_2^2+g_1^2)(\phi^{ab}+\mathcal{H}^{ab})^2-2g_1^2(\phi^{a3}+\mathcal{H}^{a3})^2\right], \end{aligned} \tag{3.4}$$

where we have ignored a constant term, expanded to second order in Φ and fourth order in h, and set $\Lambda = 4\pi f$. There are two important points to make about this result. First of all, there is a sign difference between the mass terms for the singlet and triplets. Thus, the gauge interactions introduce a saddle point instability in the vacuum. This is expected since the gauge groups would prefer the Σ vev to be proportional to the identity; at this vacuum, no gauge groups are broken and indeed the negative mass squared for the singlet attempts to rotate the vev to this non-breaking vacuum. However, as we will see later, the top sector gives equal sign contributions to both mass terms. Also from the point of view of the effective field theory, operators can be written down that give equal sign contributions to both masses or even just to the singlet. The second thing to note about the gauge contribution is that only the gauged U(1) introduces explicit custodial SU(2) violation into the potential. As a matter of fact, this will be the only interaction that can give the triplets a custodial SU(2) violating vev. Since g_1 will be approximately equal to the standard model hypercharge coupling, the triplet vevs usually give suitably small contributions to ρ. We will analyze the triplet vevs in greater detail in section 4.

Now, analyzing the top sector, we find the contribution

$$V_{\text{fermion}} = 3|y_1|^2 f^2\left[(\phi^0+\mathcal{H}^0)^2+(\phi^{ab}+\mathcal{H}^{ab})^2\right], \tag{3.5}$$

where again we have ignored a constant piece and set $\Lambda = 4\pi f$. As noted earlier, the fermion sector gives equal sign contributions to singlet and triplet masses and does not introduce custodial SU(2) breaking at this order.[3] Following the SU(6)/Sp(6) little higgs [6], we could also extend the top sector with an interaction that preserves the $\mathrm{SO}(5)_2$ symmetry. This would have the added benefit of giving equal sign contributions to the $(\phi-\mathcal{H})^2$ terms and could lift the saddle point into a local minimum. Another way to do this is through operators such as

$$\mathcal{L}_1 = a_1 f^2\sum_{i=1,j=1}^{4,4}\Sigma_{ij}\Sigma_{ij} = a_1 f^2\left[(\phi^0-\mathcal{H}^0)^2+(\phi^{ab}-\mathcal{H}^{ab})^2\right] \tag{3.6}$$

or

$$\mathcal{L}_2 = a_2 f^2\left(\sum_{i=1}^{4}\Sigma_{ii}\right)^2 = 4a_2 f^2\left[(\phi^0-\mathcal{H}^0)^2\right] \tag{3.7}$$

[3]If we had chosen $\vec{q}_t$ to contain two doublets, there would be no custodial SU(2) breaking at any order.

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which respect the $\mathrm{SO}(5)_2$ symmetry and give contributions to both singlet and triplet masses or just masses for the singlets. Depending on the UV completion of the model, these operators can be generated; for instance, they might appear naturally in an extended technicolor like completion.

These radiative corrections tell us that we must put in these operators with coefficients of their natural size of the form

$$\begin{aligned} V = \; & \lambda_{\mathbf{1}}^{-} f^2 (\phi^0 - \mathcal{H}^0)^2 + \lambda_{\mathbf{1}}^{+} f^2 (\phi^0 + \mathcal{H}^0)^2 + \\ & + \lambda_{\mathbf{3}}^{-} f^2 (\phi^{ab} - \mathcal{H}^{ab})^2 + \lambda_{\mathbf{3}}^{+} f^2 (\phi^{ab} + \mathcal{H}^{ab})^2 + \Delta\lambda_{\mathbf{3}} f^2 (\phi^{a3} + \mathcal{H}^{a3})^2 . \end{aligned} \tag{3.8}$$

As mentioned before, since g_1 will be small, $\Delta\lambda_{\mathbf{3}} \ll \lambda_{\mathbf{3}}^{\pm}$ is expected, which leads to approximately custodial SU(2) preserving triplet vevs. We will assume that the singlet and triplet masses are positive; integrating out these heavy particles then leads to a quartic coupling of the higgs (ignoring $\Delta\lambda_{\mathbf{3}}$ for simplicity):

$$\lambda |h|^4 \qquad \text{where} \qquad 4\lambda = \lambda_{\mathbf{1}} + 3\lambda_{\mathbf{3}} \tag{3.9}$$

and we've defined $1/\lambda_{(\mathbf{1},\mathbf{3})} = 1/\lambda_{(\mathbf{1},\mathbf{3})}^{-} + 1/\lambda_{(\mathbf{1},\mathbf{3})}^{+}$. Requiring a positive order one λ puts some mild constraints on the $\lambda_{(\mathbf{1},\mathbf{3})}^{\pm}$ parameters.

3.2 Log contributions to the mass parameter

Even though the little higgs mechanism protects the higgs from one-loop quadratic divergences, there are finite, one loop logarithmically divergent, and two loop quadratically divergent mass contributions, all of the same order of magnitude. Here we will analyze the logarithmically enhanced pieces as given by the one loop log term in the Coleman-Weinberg potential

$$V_{\text{one loop log}} = \frac{1}{64\pi^2} \operatorname{Str}\left[(M^\dagger M)^2 \ln \frac{M^\dagger M}{\Lambda^2} \right] . \tag{3.10}$$

The gauge contribution to the mass squared is positive

$$m^2_{\text{gauge}} = \frac{3}{64\pi^2} \left[3g^2 m^2_{W'} \ln \frac{\Lambda^2}{m^2_{W'}} + g'^2 m^2_{B'} \ln \frac{\Lambda^2}{m^2_{B'}} \right] \tag{3.11}$$

but the fermion contribution is negative

$$m^2_{\text{fermion}} = -\frac{3|y_t|^2}{8\pi^2} m^2_{t'} \ln \frac{\Lambda^2}{m^2_{t'}} , \tag{3.12}$$

where we have defined $m^2_{t'} = (|y_1|^2 + |y_2|^2) f^2$.[4] This large top contribution generically dominates and drives electroweak symmetry breaking. We've chosen not to consider the scalar contribution since it depends on the specifics behind the generation of the potential (eq. (3.8)). However, we mention that it is typically positive and subdominant to the fermion contribution.

[4]The heavy t' quark is the only heavy quark whose mass shifts when the higgs vev is turned on. Thus, it is the new heavy state that appears and cuts off the top yukawa quadratic divergence, which is also why the fermionic contribution to the higgs mass parameter only depends on $m_{t'}$.

4. Precision electroweak observables

Now that we have described the model's content and interactions, the contributions to electroweak observables can be calculated. In general, we will work to leading order in $O(v^2/f^2)$ and neglect any higher order effects. First in section 4.1, we will focus on non-oblique corrections to electroweak fermion currents and four-fermi interactions. We will demonstrate how the limit of strong g_L, g_R coupling is a "near-oblique" limit as discussed recently in [21]. This limit validates the usefulness of an S and T analysis and in sections 4.2 and 4.3 we will calculate the model's contributions to these parameters. We will choose to keep the S and T contributions from the Higgs sector, but will subtract out all other standard model contributions. Finally in section 4.4, the results of the full S-T analysis will be presented.

4.1 Electroweak currents

First of all, there are non-oblique corrections due to the exchange of the heavy gauge bosons. Specifically, integrating out the heavy vectors generates four Fermi operators and Higgs-Fermi current current interactions (the Higgs-Higgs interactions give oblique corrections and will be considered in the higgs contribution to T in section 4.2). The former are constrained by tests of compositeness and the latter after electroweak symmetry breaking induce corrections to standard model fermionic currents which are constrained by Z-pole observables. As pointed out recently by Gregoire, Smith, and Wacker [21], the S and T analysis is reliable when there exists a "near-oblique" limit where most of the non-oblique corrections vanish. The limit is called "near-oblique" since third generation quark physics still has non-vanishing effects. In this model this limit turns out to be the strong $g_L, g_R \to \infty$ limit that decouples the light generations from the heavy gauge bosons. A discussion of the non-decoupling third generation effects is outside the scope of this paper and thus they will not be analyzed. However, for a preliminary discussion of the important operators in such an analysis, see reference [21].

To calculate these induced effects, we first write down the relevant currents to the heavy gauge bosons starting with the higgs (leaving off Lorentz indices for readability)

$$\begin{aligned} j^a_{W'H} &= g\cot 2\theta j^a_H = \frac{g\cos 2\theta}{2\sin 2\theta}\, i h^\dagger \sigma^a \overleftrightarrow{D} h \\ j_{B'H} &= g'\cot 2\theta' j_H = -\frac{g'\cos 2\theta'}{2\sin 2\theta'}\, i h^\dagger \overleftrightarrow{D} h \end{aligned} \tag{4.1}$$

and also for the standard model fermions (aside from the third generation quarks)

$$j^a_{W'F} = g\tan\theta\, j^a_F \qquad j_{B'F} = g'\tan\theta'\, j_F\,, \tag{4.2}$$

where they are given in terms of the standard model $\mathrm{SU}(2)_W, \mathrm{U}(1)_Y$ currents $j^a_{(HF)}$ and $j_{(HF)}$. The one heavy gauge boson current left out is the higgs current to $W^{r\,\pm}$, but since there is no corresponding fermionic current, integrating out $W^{r\,\pm}$ does not generate four-fermi operators or standard model current corrections.

Integrating out the heavy gauge bosons generates the Higgs-Fermi interactions

$$\begin{aligned}\mathcal{L}_{\mathrm{H\ F}} &= -\frac{j^a_{\mu W'\mathrm{H}}\, j^{\mu a}{}_{W'\mathrm{F}}}{M^2_{W'}} - \frac{j_{\mu B'\mathrm{H}}\, j^{\mu}{}_{B'\mathrm{F}}}{M^2_{B'}} \\ &= -\frac{\sin^2\theta\cos 2\theta}{2f^2} j_{\mathrm{H}}{}^{a\,\mu} j_{\mathrm{F}\,a\,\mu} - \frac{\sin^2\theta'\cos 2\theta'}{2f^2} j_{\mathrm{H}}{}^{\mu} j_{\mathrm{F}\,\mu}\end{aligned} \tag{4.3}$$

and the four Fermi interactions

$$\begin{aligned}\mathcal{L}_{\mathrm{F\ F}} &= -\frac{(j^a_{\mu W'\mathrm{F}})^2}{2M^2_{W'}} - \frac{(j_{\mu B'\mathrm{F}})^2}{2M^2_{B'}} \\ &= -\frac{\sin^4\theta}{2f^2} j_{\mathrm{F}}{}^{a\,\mu} j_{\mathrm{F}\,a\,\mu} - \frac{\sin^4\theta'}{2f^2} j_{\mathrm{F}}{}^{\mu} j_{\mathrm{F}\,\mu}\,.\end{aligned} \tag{4.4}$$

As a rough guide, these operators have to be suppressed by about $(4\,\mathrm{TeV})^2$ to be safe [21, 25]. To simplify the analysis, we will take the SO(4) symmetric limit $g_L = g_R$. In this restricted case, in order to get the correct $\sin\theta_W$ requires the relation $\sqrt{3}\theta' \approx \theta$ at small θ's. Thus, the SU(2) operators give the tightest bound. Of these, the Higgs-Fermi SU(2) operator turns out to be the most constrained giving a constraint

$$m_{W'} \gtrsim 1.8\,\mathrm{TeV}\,. \tag{4.5}$$

For the value $f = 700\,\mathrm{GeV}$, this corresponds to a limit $\theta \lesssim 1/4$. However, to be safe we'll later take as a benchmark value $\theta' = 1/5\sqrt{3}, \theta \approx 1/5$ from which to compare with experiment. Note that for this near-oblique limit to exist, it was crucial that the light generations could be decoupled from the heavy gauge bosons. Again, only in this limit is an analysis of the oblique corrections S and T meaningful.

4.2 Custodial SU(2)

Custodial SU(2) violating effects are highly constrained by precision electroweak tests and this model's primary motivation is to minimize any such violation. Custodial SU(2) violation is conveniently parameterized by corrections to the ρ parameter (or equivalently the T parameter). In a little higgs model, there are potentially five sources of custodial SU(2) violation. The first possible contribution is that of expanding out the kinetic term in terms of the higgs field. The non-linear sigma model kinetic term contains interactions at high order that could give custodial SU(2) violating masses to the W and Z. However, in this model there is no violation at any order. This is due to the fact that the kinetic term is invariant under a global $\mathrm{SO}(4)_D$ that is broken down by the higgs vev to custodial SU(2). As a matter of fact, all terms in the expansion of the kinetic term just shift the value of the higgs vev v, which gives $\delta\rho = 0$.

Vector bosons. The second possibility is that integrating out the TeV scale gauge bosons (the $W', B', W^{r\,\pm}$) can generate a custodial SU(2) violating operator. This is typically denoted as

$$\mathcal{O}_4 = |h^\dagger D_\mu h|^2\,. \tag{4.6}$$

In all previous little higgs theories, integrating out the W' gauge bosons does not generate this operator at $O(v^2/f^2)$ and this holds true for this model as well. On the other hand, integrating out the B' and $W^{r\,\pm}$ *does* generate this operator, but with opposite sign! There is a cancelation with the total contribution

$$\delta\rho_{\text{Gauge Boson}} = -\frac{v^2}{16 f^2}\sin^2 2\theta' \,. \tag{4.7}$$

Note that as advertised this vanishes in the limit $\theta' \to 0$, which is the same limit where the standard model fermions decouple from the B'. At the benchmark values $\theta' = 1/5\sqrt{3}, f = 700\,\text{GeV}$, this gives a contribution $\text{T}_{\text{gauge}} = -.056$. One can see that the addition of the extra $W^{r\,\pm}$ gauge bosons has provided an extra suppression factor of $\sin^2 2\theta' \approx 1/20$.

Triplet Vev. The third contribution to custodial SU(2) violation comes from the triplet vevs. The key point is that the potential (eq. (3.8)) is custodial SU(2) invariant except for the $\Delta\lambda_{\mathbf{3}}$ term generated by the gauged U(1). The non-oblique corrections already prefer small θ' and thus small g_1. Therefore custodial SU(2) violation in the potential should be small, and we should expect that $\Delta\lambda_{\mathbf{3}} \ll \lambda_{\mathbf{3}}^{\pm}$. Calculating the triplet vev contribution, we find

$$\delta\rho_{\text{triplet}} = \frac{v^2}{16 f^2}\left[\left(\frac{\lambda_{\mathbf{3}}^- - \lambda_{\mathbf{3}}^+ - \Delta\lambda_{\mathbf{3}}}{\lambda_{\mathbf{3}}^- + \lambda_{\mathbf{3}}^+ + \Delta\lambda_{\mathbf{3}}}\right)^2 - \left(\frac{\lambda_{\mathbf{3}}^- - \lambda_{\mathbf{3}}^+}{\lambda_{\mathbf{3}}^- + \lambda_{\mathbf{3}}^+}\right)^2\right] \approx \frac{v^2}{4f^2}\frac{\lambda_{\mathbf{3}}^-(\lambda_{\mathbf{3}}^+ - \lambda_{\mathbf{3}}^-)}{(\lambda_{\mathbf{3}}^- + \lambda_{\mathbf{3}}^+)^3}\Delta\lambda_{\mathbf{3}}\,, \tag{4.8}$$

where we have expanded to first order in $\Delta\lambda_{\mathbf{3}}/\lambda_{\mathbf{3}}^{\pm}$ to get the end result. In comparison with the "littlest higgs", there is now a beneficial $\Delta\lambda_{\mathbf{3}}/\lambda_{\mathbf{3}}^{\pm}$ suppression. We cannot really say anything more in the effective field theory since there are unknown order one factors in the relation between the $\lambda_{\mathbf{3}}$'s and the coefficients as calculated in the Coleman-Weinberg potential. However, to get a feel for the expected size of the contribution, we can take the Coleman-Weinberg coefficients at face value which for $y_1 = 2$, $g_L = g_R$, and $f = 700\,\text{GeV}$, gives the plot T vs. θ' as shown in figure 1. In the limit $g_L = g_R$, there is an upper bound $\theta' \lesssim \pi/5$ (in order to get the correct $\sin\theta_W$) which is why the graph is cut off on the right. Order one factors aside, it is obvious that the triplet contribution to T is negligibly small due to the extra suppression described above.

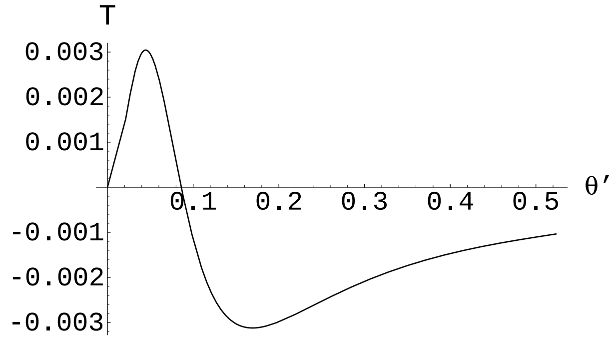

Figure 1: Triplet Contribution to T as a function of θ' with $y_1 = 2$, $g_L = g_R$, and $f = 700\,\text{GeV}$.

Top sector. The fourth source of custodial SU(2) violation is the introduction of new fermions in the top sector. To calculate the effects of the extra fermions, it is easiest to

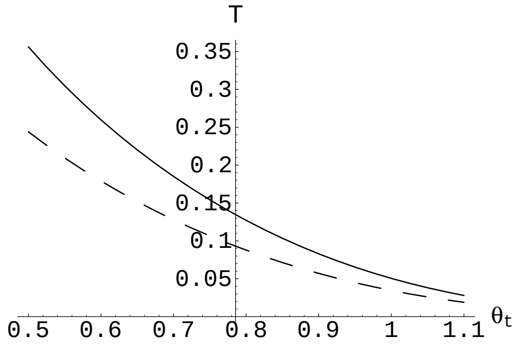

Figure 2: Top Sector Contribution to T as a function of θ_t. The solid line is for $f = 700\,\mathrm{GeV}$ while the dashed line is for $f = 900\,\mathrm{GeV}$.

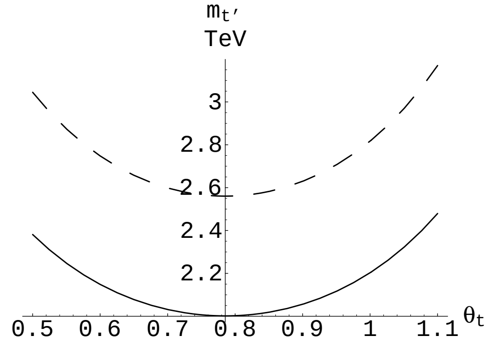

Figure 3: The heavy top mass $m_{t'}$ as a function of θ_t. Again the solid line is for $f = 700\,\mathrm{GeV}$ while the dashed line is for $f = 900\,\mathrm{GeV}$.

compute the contributions to T through vacuum polarization diagrams by the definition

$$\mathrm{T} \equiv \frac{e^2}{\alpha \sin^2\theta_W \cos^2\theta_W m_Z^2}\left[\Pi_{11}(0) - \Pi_{33}(0)\right]. \tag{4.9}$$

If we ignore the small mixing effects induced by the b quark mass, the contribution is parameterized by the single parameter θ_t where

$$y_1 = \frac{\sqrt{2}y_t}{\sin\theta_t} \qquad y_2 = \frac{\sqrt{2}y_t}{\cos\theta_t}. \tag{4.10}$$

In figure 2, plots of the T contribution versus θ_t are plotted for $f = 700\,\mathrm{GeV}$ and $f = 900\,\mathrm{GeV}$, centered around $\theta_t = \pi/4$. Note that the standard model contribution to T has already been subtracted off from the total top sector contribution, in order to give the final plotted results. A good fit to the T contribution in the range of θ_t plotted is $\cos^2\theta_t \cot\theta_t$, where the fit gets bad in the $\theta_t \leq 1/2$ region. As we change f, the constant of proportionality roughly scales as $1/f^2$. In figure 3, the dependence of $m_{t'}$ on θ_t is also plotted. An important point is that naturalness puts an upper bound constraint on the mass $m_{t'}$. By the standard given in [5], for a 200 GeV Higgs, 10% fine-tuning restricts $m_{t'} \lesssim 2\,\mathrm{TeV}$. Fortunately, as the figures show, it appears possible to get corrections to T within the 1-σ bound at f scales consistent with this amount of fine tuning. It is also important to keep in mind that these T contributions are quite mild (this appears to be a generic feature of little higgs models). For instance, the standard model top quark contribution is $\mathrm{T} \approx 1.2$ which is quite larger than the largest value in the plot of 0.35. It is also well known that moderate positive values of T increase the upper bound on the higgs mass [26]. Since θ_t will be varied during the fit, this will dramatically change the allowed higgs masses.

Higgs. Finally, the higgs itself will contribute to T. The contribution is well known and we will use the explicit formula contained in [27]. For the purposes of this paper, we will take the S,T origin when $m_{\text{higgs, ref}} = 115\,\mathrm{GeV}$. As we increase the higgs mass, this T contribution gets large and negative.

4.3 S parameter

The S parameter along with the T parameter gives a good handle on the oblique contributions of any new physics. To the order at which we have been calculating (i.e. v^2/f^2), there are only two sources of S contributions. The first contribution is that of the higgs. Again, we use the result in [27]. This gives a positive S contribution for higgs masses larger than our chosen reference mass.

The second contribution to S comes from the extended top sector and again is best calculated via vacuum polarization diagrams using the definition

$$S \equiv -16\pi\Pi'_{3Y}(0)\,. \tag{4.11}$$

In figure 4, we have plotted the beyond the standard model S contribution from the top sector for $f = 700$ GeV. For the region of naturalness, the contributions to S are quite small and do not measurably affect the fit of the model.

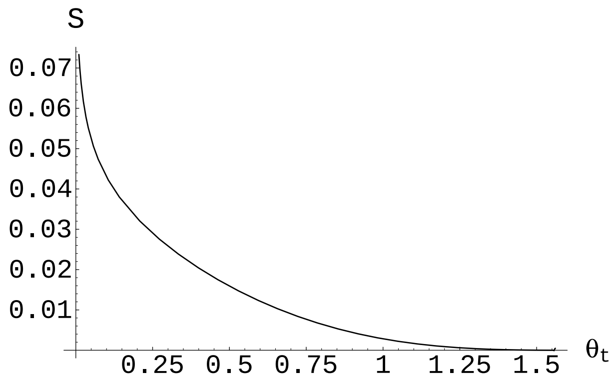

Figure 4: Top Sector Contribution to S for $f = 700\,\text{GeV}$ as a function of θ_t.

4.4 Summary of limits

Now, the fit to S and T can be performed. In figure 5, the approximate 1σ ellipse in the S-T plane as given in [28] has been plotted. Note that the (S=0,T=0) origin has been set to the reference values $m_h = 115\,\text{GeV}$ and $m_t = 174.3\,\text{GeV}$. Sloping down and to the right, the exact contribution due to the higgs has been plotted for the masses $m_h = (115, 200, 300, 500, 700, 1000)\,\text{GeV}$. To represent the other two contributions, two points of the beyond the standard model fermion and gauge contributions have also been plotted for the values $\theta_t = \pi/4$, $\theta' = 1/5\sqrt{3}$, and $f = 700, 900\,\text{GeV}$. The fermionic contribution generically points up and slightly to the right whereas the gauge contribution points downward. To find where the model is on the S-T plane, these three contributions should be added.

To be specific, we'll focus on the value $f = 700\,\text{GeV}$ as this limits the amount of fine-tuning in the model. In figure 6, the S and T contributions for the higgs and fermions are summed for $\theta_t = \pi/4$ and $\theta' = 1/5\sqrt{3}$. From the graph there appears to be a generous range of m_h that falls within the 1σ limits, at least $115\,\text{GeV} \leq m_h \lesssim 400\,\text{GeV}$. If θ_t is changed, larger higgs mass can be attained. Although changing θ_t from the equal mixing value $\pi/4$ increases fine-tuning, as m_h increases the higgs mass parameter also increases which will reduce the fine-tuning, and thus θ_t can be manipulated as the higgs gets heavier. This freedom helps since reducing θ_t will increase the fermion contribution to T (and only slightly increase S) as required to stay within the ellipse [26]. For instance, at $\theta_t = \pi/6$, we can still tolerate a $1\,\text{TeV}$ higgs mass. Changing θ' produces less of an effect, but as it goes to zero, it can also help improve the fit at large higgs mass. Of course, at higgs

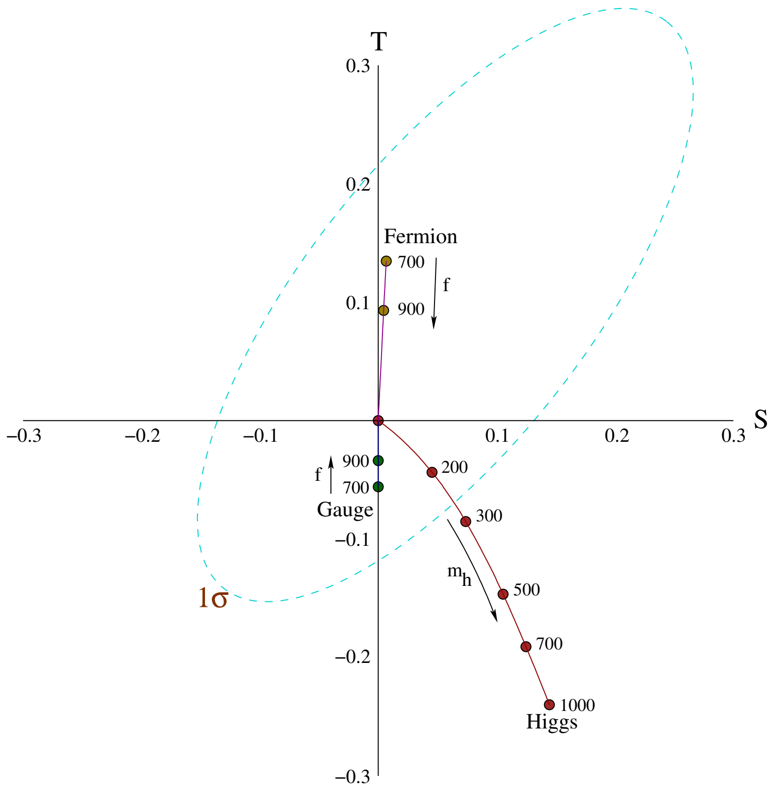

Figure 5: The approximate 1σ ellipse in the S-T plane. The origin (S=0,T=0) corresponds to the reference values $m_h = 115\,\text{GeV}$ and $m_t = 174.3\,\text{GeV}$. The higgs contribution for increasing higgs mass is plotted for the values $(115, 200, 300, 500, 700, 1000)\,\text{GeV}$ which slopes down and to the right. Two representative points of the extra fermion and gauge contributions have also been plotted for $\theta_t = \pi/4$, $\theta' = 1/5\sqrt{3}$ and $f = 700, 900\,\text{GeV}$.

masses about a TeV, the little higgs mechanism is not even required if the cutoff is taken to be 10 TeV. However, within our model, we see that a large region of parameter space is allowed by both the S-T fit and naturalness. In figure 7, θ' and f have been fixed while θ_t and m_h are scanned; all points in the shaded region fit within the 1σ S-T ellipse, while all points above a dashed line are consistent with that percentage of fine-tuning (again using the fine-tuning definition of [5]). There is quite a large range of higgs masses allowed by precision constraints, and most of it is within ten percent fine-tuning or better.

As a brief comment on more general f values, the positive fermion contribution to T decreases as f increases at a given θ_t, so it is more difficult to get within the ellipse for very large higgs mass at large f. For instance, going up to $f = 900\,\text{GeV}$ pushes the range for $\theta_t = \pi/4$ and $\theta' = 1/5\sqrt{3}$ down to about $115\,\text{GeV} \leq m_h \lesssim 350\,\text{GeV}$. However, it is our hope that naturalness will help to keep f low, so that the TeV scale particles can still be discovered at the LHC.

Two more comments on this fit should be made. First of all, the experimental error in the top mass gives an uncertainty in the standard model contribution to S and T. With the current error of $\pm 5\,\text{GeV}$, this introduces an unknown $\pm .07$ contribution to T (the change in S is small), which can significantly affect the fit. The other thing to note is

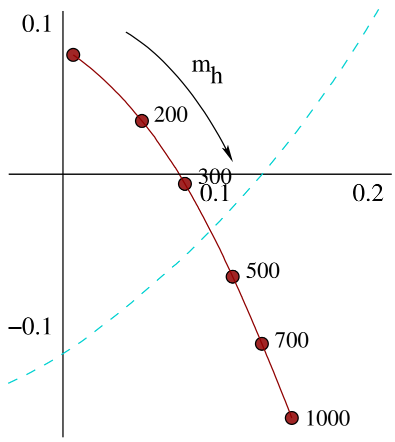

Figure 6: A closeup of the S-T plot with the summed contributions of the higgs, gauge bosons, and fermions. In this plot, the values $f = 700\,\mathrm{GeV}$, $\theta_t = \pi/4$, $\theta' = 1/5\sqrt{3}$ are fixed, but higgs mass is allowed to vary.

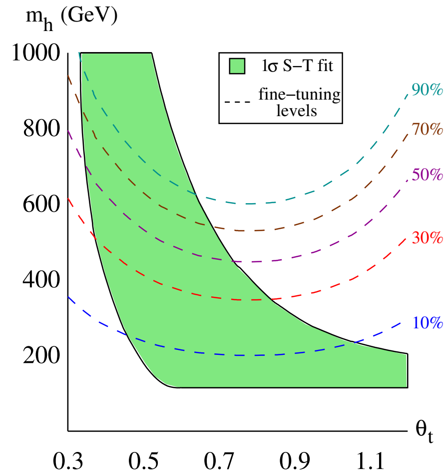

Figure 7: A scan of m_h vs. θ_t with the fixed values $\theta' = 1/5\sqrt{3}$ and $f = 700\,\mathrm{GeV}$. The shaded region is allowed by the 1σ fit while points above a dashed line are consistent with that percentage of fine-tuning.

to remember that $O(v^4/f^4)$ effects and higher have been neglected. For instance, as seen in [21], S contributions from $O(v^4/f^4)$ and dimension 6 operators suppressed by Λ^2 are of the order $\pm.02$. Thus, to go beyond the $O(v^2/f^2)$ analysis as presented here will require some assumptions about the UV completion.

As a conclusion to this section, we plot some sample spectrums for an allowed region of parameter space in figure 8, with $f = 700\,\mathrm{GeV}$. For the heavy quark sector, we have allowed θ_t to vary. The t' quark is the heavy charge 2/3 quark that cuts off the top yukawa quadratic divergence, and is nearly degenerate with the charge $-1/3$ b' quark (the b' is usually about $5 - 10\,\mathrm{GeV}$ heavier). The T and Ψ quark (charge 2/3 and 5/3 respectively) are exactly degenerate at tree level and are not important in cutting off the top quadratic divergence. In general, the (T, Ψ) pair is lighter than the (t', b') pair. In the heavy gauge boson sector, the W' is generally the heaviest and the B' and $W_r^{\pm}$ are nearly degenerate (with the B' heavier). As θ decreases, all of the states get heavier and more degenerate. Finally, in the scalar sector, the simplifying assumption that naturalness in the gauge contribution sets $\lambda_{\mathbf{1}}^{\pm} = 3\lambda_{\mathbf{3}}^{\pm}$ has been assumed. This naturalness condition sets the scalars to be heavier than the triplets by a factor of $\sqrt{3}$. To simplify it even further, we've also assumed that $\lambda_{\mathbf{3}}^{-} = \lambda_{\mathbf{3}}^{+}$ and picked a higgs mass of $200\,\mathrm{GeV}$. All these particles have TeV scale masses and should be searched for at the LHC.

5. Conclusion

In this paper, a new "littlest higgs" model with custodial SU(2) symmetry has been analyzed. Precision electroweak analyses of little higgs models has given suggestions on what features little higgs theories should realize, and with this motivation, the $\frac{\mathrm{SO}(9)}{\mathrm{SO}(5)\times\mathrm{SO}(4)}$ model has been proposed in order to be easily compatible with precision constraints. Some of the

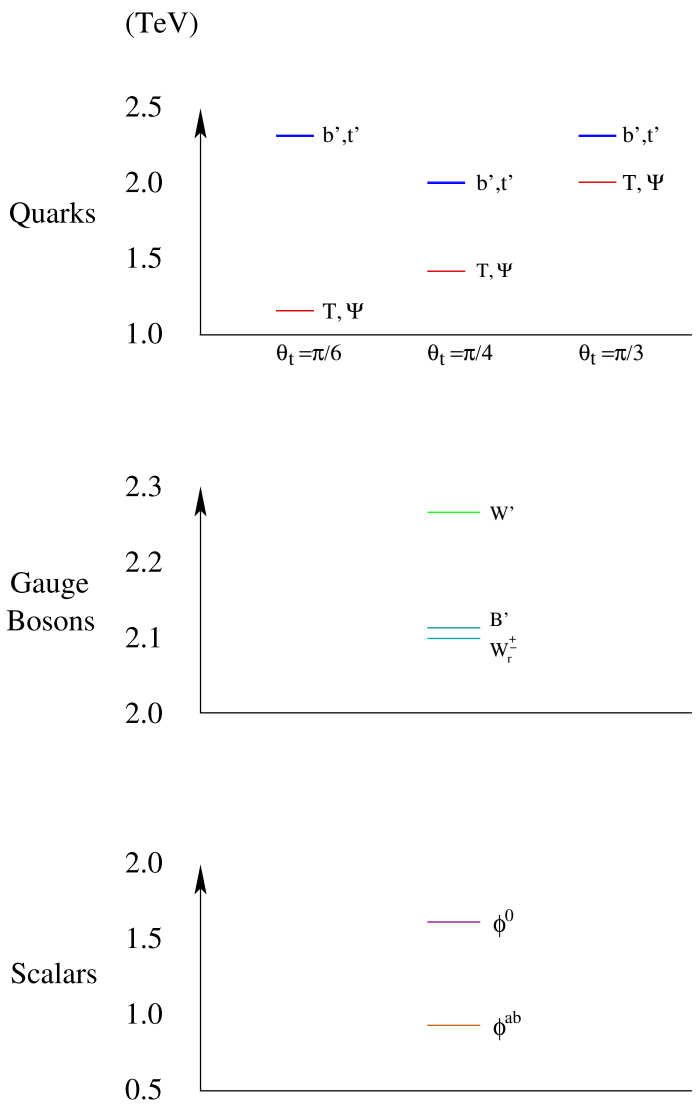

Figure 8: A sample spectrum for the values $f = 700\,\text{GeV}, \theta = 1/5, \theta' = 1/(5\sqrt{3})$. In the scalar sector, $m_h = 200\,\text{GeV}$ and $\lambda_{\mathbf{1}}^{\pm} = 3\lambda_{\mathbf{3}}^{\pm} = 4/3$ has been assumed.

unique features of the model include:

- Psuedo-goldstone bosons with custodial SU(2) preserving vevs, comprised of a single light higgs doublet and at the TeV scale, a singlet and three $\text{SU}(2)_W$ triplets, similar to [24].
- At $O(v^2/f^2)$, the 1σ S-T fit allows a generous range of higgs masses in a large region of parameter space that is consistent with naturalness.

The other features follow that of the original “littlest higgs”, including the generation of the higgs potential through gauge and fermion interactions as well as the fermion sector driving electroweak symmetry breaking. At low energies, the effective theory is the standard model, with extra states at the TeV scale to cut off the quadratic divergences to the higgs. Once again, we emphasize that the precision constraints are mild and a large region of higgs masses is allowed in parameter space where the higgs mass is natural.

In analyzing the one loop quadratically divergent term in the Coleman-Weinberg potential, we discovered that the gauge interactions introduced a saddle point instability in the vacuum. Two solutions to stabilize the vacuum were presented, either through extending the top sector or writing down operators that could give same sign contributions to the singlet and/or triplet masses. As an aside, we mention here briefly two "littlest higgs" models that also contain custodial SU(2) where the preferred vacuum is stable. Firstly, changing the global symmetry from SO(9) to SU(9) changes the breaking pattern to SU(9) $\rightarrow$ SO(9). In this model, the upper SU(4) gauge group can be gauged instead of SO(4). The SU(4) gauge interactions prefer the off-diagonal vacuum and stabilize the $(\phi - X)^2$ terms. This along with the top sector given earlier can stabilize the vacuum. This theory contains 2 higgs doublets, 3 singlets, and 6 triplets and should preserve custodial SU(2) in the same way as the SO(9) model.

Recently, the idea of UV completing the "littlest higgs" via strong interactions giving rise to composite fermions and composite higgs was introduced [29]. This idea requires the top sector to be comprised of full multiplets of the global symmetry. A model with custodial SU(2) symmetry that conceivably could be UV completed in this manner is one based on the coset $\frac{\mathrm{SU}(8)}{\mathrm{Sp}(8)}$, where the upper 4 components of the 8 is $4 \equiv (2_L + 2_R)$ and $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(2) \times \mathrm{U}(1)$ is gauged. This time the gauge interactions naively make the vacuum a local maximum, but the sign could depend on the UV completion and is just a discrete choice. The spectrum of this theory turns out to be 4 higgs doublets and 5 singlets! However, as one can see from these other examples, the model presented in this paper has the simplest spectrum, displays all the important physics, and is a complete and realistic model.

In summary, little higgs models are exciting new candidates for electroweak symmetry breaking. They contain naturally light higgs boson(s) that appear as pseudo-goldstone bosons through the breaking of an approximate global symmetry. With perturbative physics at the TeV scale, these models produce relatively benign precision electroweak corrections. In this paper, one model that realizes custodial SU(2) symmetry has been described, which may give some insight into why the standard model has worked so well for so long. In the near future, experiments at the LHC should start giving indications whether or not these candidate theories play a role in what comes beyond the standard model.

Acknowledgments

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A. Generators and notation

The SO(4) commutation relations are:

$$[T^{mn}, T^{op}] = \frac{i}{\sqrt{2}}(\delta^{mo}T^{np} - \delta^{mp}T^{no} - \delta^{no}T^{mp} + \delta^{np}T^{mo}), \tag{A.1}$$

where m, n, o, p run from $1, \ldots, 4$. These generators can be broken up into

$$T^{l\,a} = \frac{1}{2\sqrt{2}}\epsilon^{abc}T^{bc} + \frac{1}{\sqrt{2}}T^{a4} \qquad T^{r\,a} = \frac{1}{2\sqrt{2}}\epsilon^{abc}T^{bc} - \frac{1}{\sqrt{2}}T^{a4}, \tag{A.2}$$

where a, b, c run from $1, \ldots, 3$. The commutation relations in this basis of SO(4) are equivalent to $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$:

$$[T^{l\,a}, T^{l\,b}] = i\epsilon^{abc}T^{l\,c}, \qquad [T^{r\,a}, \tau^{r\,b}] = i\epsilon^{abc}T^{r\,c}, \qquad [T^{l\,a}, T^{r\,b}] = 0\,. \tag{A.3}$$

Vector representation. The vector representation of SO(4) can be realized as:

$$T^{mn\;op} = \frac{-i}{\sqrt{2}}(\delta^{mo}\delta^{np} - \delta^{no}\delta^{mp}), \tag{A.4}$$

where m, n, o, p again run over $1, \ldots, 4$ and m, n label the SO(4) generator while o, p are the indices of the vector representation. In this representation:

$$\mathrm{Tr}\; T^A T^B = \delta^{AB}\,. \tag{A.5}$$

Higgs representations. For the higgs doublet, we have three equivalent forms of the representation. First, there is the SO(4) vector representation, denoted as

$$\vec{h} \equiv \begin{pmatrix} h^a \\ h^4 \end{pmatrix} \qquad \text{where } a = 1, 2, 3, \tag{A.6}$$

the $\mathrm{SU}(2)_W$ doublet representation (with $\mathrm{Y} = -1/2$)

$$h \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} h^4 + ih^3 \\ -h^2 + ih^1 \end{pmatrix} \tag{A.7}$$

and the two by two matrix

$$H \equiv \frac{(h^4 + \sigma^a\, h^a)}{\sqrt{2}} = \begin{pmatrix} h & -\epsilon h^* \end{pmatrix}, \tag{A.8}$$

where the antisymmetric tensor $\epsilon = i\sigma^2$. Under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, this matrix transforms as

$$H \to L\, H\, R^\dagger \tag{A.9}$$

and thus a vev in the h^4 direction breaks $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ to custodial SU(2).

The Coleman-Weinberg potential depends on the fields $\mathcal{H}^0$ and $\mathcal{H}^{ab}$ as defined by $\frac{1}{2f}\vec{h}\,\vec{h}^{\,T} = \mathcal{H}^0 + 4\mathcal{H}^{ab}\, T^{l\,a}T^{r\,b}$. These are given in terms of the higgs fields as:

$$\mathcal{H}^0 = \frac{1}{4f}|h|^2 \qquad \mathcal{H}^{ab} = \frac{1}{8f}\left[\left(h^c h^c - h^4 h^4\right)\delta^{ab} - 2h^a h^b - 2\epsilon^{abc}h^c h^4\right] \tag{A.10}$$

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Singlet and triplet representations. In this theory, there are TeV scale scalars transforming as a singlet and as triplets under $\mathrm{SU}(2)_W$, which appear in the symmetric product of two SO(4) vectors, i.e. $(\mathbf{4}\times\mathbf{4})_S = \mathbf{1}+\mathbf{9} = (\mathbf{1}_L,\mathbf{1}_R)+(\mathbf{3}_L,\mathbf{3}_R)$. In the non-linear sigma model field Σ, these appear in the symmetric four by four matrix Φ and can be written as

$$\Phi = \phi^0 + 4\phi^{ab}\, T^{l\,a} T^{r\,b}\,. \tag{A.11}$$

Note that since the left and right generators commute, this is a symmetric matrix. These fields are canonically normalized and for the triplets, $\mathrm{SU}(2)_W$ acts on the a index in the triplet representation and $\mathrm{U}(1)_Y$ acts on the b index by $T^{r\,3}$ in the triplet $\mathrm{SU}(2)_R$ representation.

Fermion representation. The SO(4) vector representation fits nicely for the scalars, but is a bit cumbersome for the fermion sector. However, taking inspiration from the $\vec{h}, H$ transformation properties, it isn't hard to see the correct correspondence. Let's first start with a set of doublets q_1, q_2 in a two by two matrix

$$Q \equiv (q_1\ q_2) \tag{A.12}$$

which transforms under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ as

$$Q \to L\, Q\, R^\dagger\,. \tag{A.13}$$

Thus the q's are $\mathrm{SU}(2)_L$ doublets and the $\mathrm{SU}(2)_R$ rotates them into each other. Now, Q can be transformed into an SO(4) vector by tracing

$$\vec{q}^{\,T} = (q^a, q^4) \equiv \frac{1}{\sqrt{2}}\left(\mathrm{Tr}(-i\sigma^a Q), \mathrm{Tr}(Q)\right), \tag{A.14}$$

where $\vec{q}$ transforms under the T^l, T^r generators of the SO(4) representation. The normalization out front is important if this is to be completed into an SO(5) vector by the addition of a singlet fermion Ψ (this is just in order to keep canonical normalization under group action). Finally, the generalization of this correspondence to a fermion transforming under under an $\mathrm{SU}(2)\times\mathrm{U}(1)$ gauge group is straightforward.

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